Minisymposium "Representation theory of Lie superalgebras"
September 19–21, 2011, Cologne, Germany

Organisers: A. Alldridge (Cologne) and M. Gorelik (Weizmann Institute)

Schedule

Venue: S 91, Philosophikum, Universität zu Köln, Albertus-Magnus-Platz, Cologne

Monday	**Speaker**	**Title**
14:00–14:50	Papi	*Denominator identities for finite-dimensional Lie superalgebras*
15:00–15:50	Frajria	*Superalgebras and Theta correspondence over the real numbers*
16:00–16:30	COFFEE BREAK	
16:30–17:20	Salmasian	*An analytic approach to unitary representations of Lie supergroups*
17:30–18:20	Gavarini	*Algebraic supergroups associated to simple Lie superalgebras*
Tuesday		
14:00–14:50	Mazorchuk	*Serre functors for Lie superalgebras*
15:00–15:50	Van der Jeugt	*Wigner quantization and representations of Lie superalgebras*
16:00–16:30	COFFEE BREAK	
16:30–17:20	Cheng	*Kostant homology formula for oscillator representations of classical Lie superalgebras*
17:30–18:20	Wang	*A super duality approach to the representation theory of Lie superalgebras*
Wednesday		
14:00–14:50	Serganova	*Borel-Weil-Bott theorem and Bernstein-Gelfand-Gelfand reciprocity for classical supergroups*
15:00–15:50	Stroppel	*Graded representation theory and Koszulity for* $\mathfrak{gl}(m\|n)$

Abstracts

Shun-Jen Cheng
Academia Sinica
Kostant homology formula for oscillator representations of classical Lie superalgebras

Pierluigi Möseneder Frajria
Politecnico di Milano
Superalgebras and Theta correspondence over the real numbers

Abstract: The odd part of the super denominator of a basic classical Lie superalgebra is the character of the oscillator representation of the odd part of the superalgebra. It is then natural to use superalgebra techniques to study this representation. In particular, we will show how the generalized denominator formulas for basic classical Lie superalgebras can be used to derive the Theta correspondence between representations of a compact dual pair.

Fabio Gavarini
Università degli Studi di Roma "Tor Vergata"
Algebraic supergroups associated to simple Lie superalgebras

Abstract: For any finite dimensional (complex) simple Lie superalgebra I provide an explicit recipe to construct an algebraic supergroup G (defined via its functor of points) whose tangent Lie superalgebra is $\mathfrak{g}$ itself. To do that, I generalise the classical Chevalley method, which constructs a (semi-) simple algebraic group starting from any complex, f. d. (semi-) simple Lie algebra $\mathfrak{g}$ and from a faithful f. d. $\mathfrak{g}$-module V: the basic ingredient to make use of is the datum of a so-called "Chevalley basis". I shall show that one can do the same when $\mathfrak{g}$ is replaced with a simple Lie *super*algebra: one introduces then a notion of "Chevalley basis", one proves the existence of the latter, and then one essentially implements the same method (with many new aspects to deal with, of course). A remarkable fact is that this strategy is succesful both with the (simple) Lie superalgebras of classical type and with those of Cartan type—somehow extending the range of application of Chevalley's original idea.
Besides this "existence" result, I shall present also a "uniqueness" one: every connected algebraic supergroup whose Lie superalgebra be (f.d.) simple is isomorphic to one of the supergroups that I just constructed. Thus one eventually finds a complete classification of such supergroups.

References:

Fioresi, R., Gavarini, F. (2011). Chevalley Supergroups. *Mem. Amer. Math. Soc.*, in press. arXiv:0808.0785.
Gavarini, F. (2008). Chevalley Supergroups of type $D(2,1;a)$. arXiv:1008.1838.
Fioresi, R., Gavarini, F. (2010). On the construction of Chevalley Supergroups. In: Supersymmetry in Mathematics and Physics, *Lect. Notes Math.*, to appear. arXiv:1008.1838.
Fioresi, R., Gavarini, F. Algebraic supergroups with classical Lie superalgebras, in preparation.
Gavarini, F. Algebraic supergroups of Cartan type, in preparation.

Crystal Hoyt
Bar-Ilan University
Good gradings of basic Lie superalgebras

Abstract: A finite-dimensional simple Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ is called basic if $\mathfrak{g}_{\bar{0}}$ is a reductive Lie algebra and there exists a non-degenerate even invariant bilinear form on $\mathfrak{g}$. These are the Lie superalgebras: $\mathfrak{sl}(m|n) : m \neq n$, $\mathfrak{psl}(n|n)$, $\mathfrak{osp}(m|2n)$, $F(4)$, $G(3)$ and $D(2,1,\alpha)$. A $\mathbb{Z}$-grading $\mathfrak{g} = \oplus_{j\in\mathbb{Z}}\mathfrak{g}(j)$ is called good if there exists an element $e \in \mathfrak{g}_{\bar{0}}(2)$ such that the map ad $e : \mathfrak{g}(j) \to \mathfrak{g}(j+2)$ is injective for $j \leq -1$ and surjective for $j \geq -1$. For example, if $e \in \mathfrak{g}_{\bar{0}}$ belongs to an $\mathfrak{sl}_2$-triple $\{e, f, h\}$ where $[e,f] = h$, $[h,e] = 2e$ and $[h,f] = -2f$, then the $\mathbb{Z}$-grading of $\mathfrak{g}$ given by the eigenspaces of ad h is a good $\mathbb{Z}$-grading for e, and is called a Dynkin grading.

Good $\mathbb{Z}$-gradings of finite-dimensional simple Lie algebras were classified by V.G. Kac and A.G. Elashvili in 2005. This problem arose in connection to W-algebras, where good $\mathbb{Z}$-gradings play a role in their construction. We will discuss the proof of the classification of good $\mathbb{Z}$-gradings for the basic Lie superalgebras, which involves determining the centralizers of nilpotent even elements and of sl(2)-triples in basic Lie superalgebras. We will show that all good $\mathbb{Z}$-gradings of the exceptional Lie superalgebras: $F(4)$, $G(3)$,and $D(2,1,\alpha)$ are Dynkin gradings. The good $\mathbb{Z}$-gradings of $\mathfrak{sl}(m|n)$: $m \neq n$, $\mathfrak{psl}(n|n)$ and $\mathfrak{osp}(m|2n)$ are classified using certain combinatorial objects called "pyramids", analogously to the Lie algebra setting. We will also discuss the relationship between good even gradings and the existence of Richardson elements in parabolic subalgebras.

Volodymyr Mazorchuk
Uppsala University
Serre functors for Lie superalgebras

Abstract: In this talk I will try to show how one can use Harish-Chandra bimodules to describe Serre functors on the BGG category $\mathcal{O}$ for certain finite dimensional Lie (super)algebras. Although there are various descriptions of Serre functors for Lie algebras, our description using Harish-Chandra bimodules is new even in this classical case. As a consequence, we show that in "good" cases the algebra describing the category of finite dimensional modules for a Lie superalgebra is symmetric. (Joint work with Vanessa Miemietz.)

Paolo Papi
Università di Roma "La Sapienza"
Denominator identities for finite-dimensional Lie superalgebras

Abstract: This is a joint work with M. Gorelik, P. Moseneder Frajria and V. Kac. We provide formulas for the denominator and superdenominator of a basic classical type Lie superalgebra for any set of positive roots. I'll try to explain the motivations which led us to deal with this problem (related to my previous work, joint with Moseneder and Kac, on Dirac operators), and the combinatorial setting for its solution.

Hadi Salmasian
University of Ottawa
An analytic approach to unitary representations of Lie supergroups

Abstract: In this talk I will present an overview of recent progress on the classification of unitary representations of finite and infinite dimensional Lie supergroups. In the finite dimensional case, we can show that under natural conditions all unitary representations are highest weight modules in the appropriate sense. I will also describe an analogue of the classical orbit method which applies to nilpotent Lie supergroups. In the case of infinite dimensional Lie supergroups, I will introduce a category of representations which is closed under restriction, and also contains natural modules such as the oscillator representation. Part of this talk is based on joint work with Karl-Hermann Neeb.

Vera Serganova
University of California at Berkeley
Borel-Weil-Bott theorem and Bernstein-Gelfand-Gelfand reciprocity for classical supergroups

Abstract: In 2003 J. Brundan found remarkable connections between the category F of finite-dimensional representations of $GL(m,n)$ and tensor representations of $GL(\infty)$. In the recent paper Brundan and Stroppel develop this idea further using a categorification approach and construct a certain Koszul algebra which completely describes the structure of the category F. The important ingredient of this approach is the Bernstein-Gelfand-Gelfand reciprocity law which relates projective and simple objects in F via so called Kac modules. The characters of Kac modules are easy to compute, hence knowing of multiplicities of Kac modules in projective modules allows one to obtain the character of a simple module. Unfortunately, Kac modules do not exist for other classical supergroups. We will show that BGG reciprocity holds if in place of a Kac module one takes a virtual module given by the Euler characteristic of a line bundle on a flag supermanifold. We give a combinatorial algorithm for decomposition of a projective module into a sum of these virtual modules for

the orthosymplectic supergroup and explore connections with representations of $GL(\infty)$ on this case. This talk is based on a joint work with C. Gruson.

Catharina Stroppel
Universität Bonn
Graded representation theory and Koszulity for $\mathfrak{gl}(m|n)$

Joris Van der Jeugt
Ghent University
Wigner quantization and representations of Lie superalgebras

Abstract: For many quantum systems described by a Hamiltonian involving oscillators or oscillator-like interactions, the technique of Wigner quantization leads to algebraic compatibility conditions between operators. These conditions can be solved in terms of Lie superalgebra generators, in particular in terms of the Lie superalgebras $\mathfrak{osp}(1|2n)$ and $\mathfrak{gl}(1|n)$. This implies that the (unitary) representations of these Lie superalgebras are important in order to describe some physical quantities of these systems (position spectrum, energy, angular momentum contents).

We shall focus on some examples where interesting representation theoretic questions appear. For $\mathfrak{osp}(1|2n)$ this involves the characters (and a character formula) of a class of infinite-dimensional lowest weight representations, and the decomposition of these representations with respect to (physically relevant) subalgebras. For $\mathfrak{gl}(1|n)$, this involves finite-dimensional representations (with known characters, e.g. in terms of Schur functions), and here we present some generating function techniques to describe decompositions with respect to subalgebras. In doing so, we use various computational results from the field of symmetric and supersymmetric Schur functions.

Weiqiang Wang
University of Virginia
A super duality approach to the representation theory of Lie superalgebras

Acknowledgements

Financial support from the following institutions is gratefully acknowledged:

- DFG-Schwerpunkt 1388 “Darstellungstheorie”
- SFB/Transregio 12 “Symmetries and Universality in Mesoscopic Systems”, funded by DFG
- Leibniz junior independent research group grant, funded by DFG